

Lecture 8: Performance of Communication Systems

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EE421: Communications I. For more information read Section 9.9 in your textbook or visit <http://wikipedia.org/>.

Shannon's Limit

$$C = B_{ch} \times \log_2(1 + SNR)$$

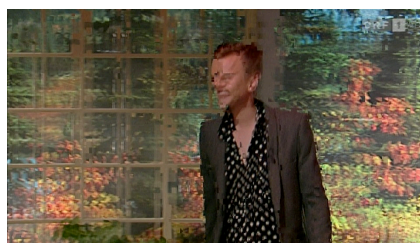
- C : Capacity of the channel in bits/second (bps)
- B_{ch} : Channel bandwidth (units of Hz)
- SNR : Signal-to-Noise Ratio (unitless)(*not* dB)

Signal-to-Noise Ratio (SNR)

$$SNR = \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{P_x}{P_n} \quad (\text{unitless})$$

- **Noise:** all random and unpredictable signals added to the message by the channel.
- External and internal sources of noise.
- Solutions exist to reduce noise power (but noise can *never* be eliminated).
- In analog systems, SNR decides the *Quality* of the received *noisy* signal.
- In digital systems, SNR decides *BER*, i.e., *Quality*.

Quality



SNR is usually expressed in dB

$$SNR = \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{P_x}{P_n} \quad (\text{unitless})$$

$$SNR = 10 \times \log_{10} \left(\frac{P_x}{P_n} \right) \quad (\text{dB})$$

$$\frac{P_2}{P_1} [\text{dB}] = 10 \times \log_{10} \left(\frac{P_2}{P_1} [\text{unitless}] \right) = 10 \times \log_{10} \left(\left(\frac{V_2}{V_1} \right)^2 [\text{unitless}] \right)$$

$$\frac{V_2}{V_1} [\text{dB}] = 20 \times \log_{10} \left(\frac{V_2}{V_1} [\text{unitless}] \right)$$

Unitless vs. dB

$$\frac{P_2}{P_1} [\text{dB}] = 10 \times \log_{10} \left(\frac{P_2}{P_1} [\text{unitless}] \right)$$

$$\frac{P_2}{P_1} [\text{unitless}] = 10^{\left[\frac{\frac{P_2}{P_1} [\text{dB}]}{10} \right]}$$

$$1000 \text{ unitless} \Rightarrow 10 \times \log_{10}(1000) = 30 \text{ dB}$$

$$30 \text{ dB} \Rightarrow 10^{\left[\frac{30}{10} \right]} = 10^3 = 1000 \text{ unitless}$$

Gain, Power	Gain in dB
1 (no gain)	0 dB
2 (twice the power)	≈ +3 dB
10 (ten times the power)	+10 dB
100	+20 dB
1000	+30 dB
10000	+40 dB

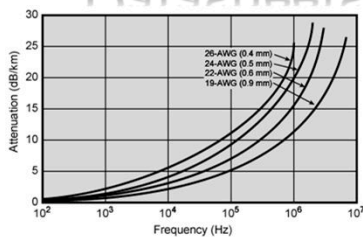
Attenuation, Power	Attenuation in dB
0.5 (half the power)	≈ -3 dB
0.25 (quarter the power)	≈ -6 dB
0.1 (tenth the power)	-10 dB
0.01 (one hundredth)	-20 dB
0.001 (one in a thousand)	-30 dB
0.0001 (one in 10 thousand)	-40 dB

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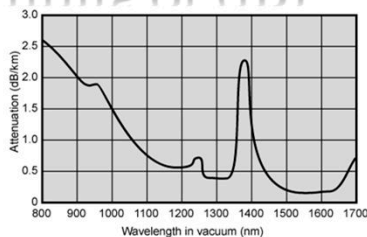
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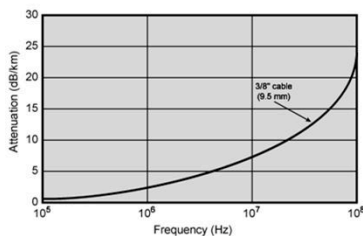
Datasheets (units of dB)



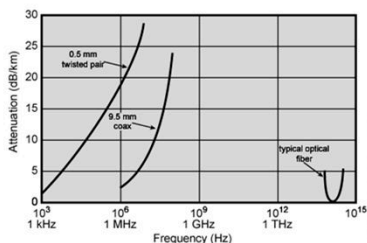
(a) Twisted pair (based on [REEV95])



(c) Optical fiber (based on [FREE02])



(b) Coaxial cable (based on [BELL90])



(d) Composite graph

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dB, dBm and dBW

$$\frac{P_2}{P_1} [\text{unitless}] \rightarrow \frac{P_2}{P_1} [\text{dB}] = 10 \times \log_{10} \left(\frac{P_2}{P_1} [\text{unitless}] \right)$$

$$P_2 [\text{mW}] \rightarrow P_2 [\text{dBm}] = 10 \times \log_{10} \left(\frac{P_2 [\text{mW}]}{1 \text{ mW}} \right)$$

$$P_2 [\text{W}] \rightarrow P_2 [\text{dBW}] = 10 \times \log_{10} \left(\frac{P_2 [\text{W}]}{1 \text{ W}} \right)$$



Easier Math!

$$\log_{10}(ab) = \log_{10}(a) + \log_{10}(b)$$

Multiplication → Addition

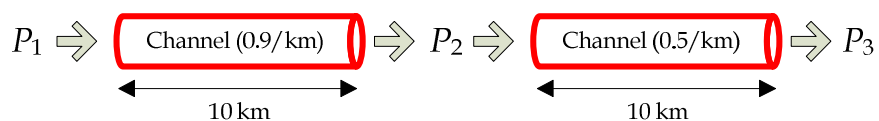
$$\log_{10} \left(\frac{a}{b} \right) = \log_{10}(a) - \log_{10}(b)$$

Division → Subtraction



Example

- Assume P_1 is 100 mW. Find P_3 in dBm.

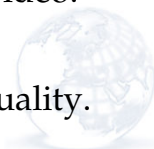


Summary

- | | |
|------------------|---|
| • dB + dB = dB | • unitless × unitless = unitless |
| • dBW + dB = dBW | • W × unitless = W |
| • dBm + dB = dBm | • mW × unitless = mW |
| • dB - dB = dB | • $\frac{\text{unitless}}{\text{unitless}} = \text{unitless}$ |
| • dBm - dB = dBm | • $\frac{\text{mW}}{\text{unitless}} = \text{mW}$ |
| • dBm - dBm = dB | • $\frac{\text{mW}}{\text{mW}} = \text{unitless}$ |
| • dBW + dBW = ? | • W × W = W^2 |
| • dBm + dBm = ? | • mW × mW = μW^2 |

SNR vs. Quality

- For **voice** signals:
 - SNR = 5 dB to 10 dB at the receiver output implies a barely intelligible signal.
 - SNR = 25 dB to 35 dB is telephone quality signal.
 - **Summary:** If SNR \geq 30 dB, good quality voice.
- For **video** signals:
 - **Summary:** If SNR \geq 50 dB, good quality video.
- For **digital** signals:
 - Need enough SNR for BER $\leq 10^{-6}$, good quality.

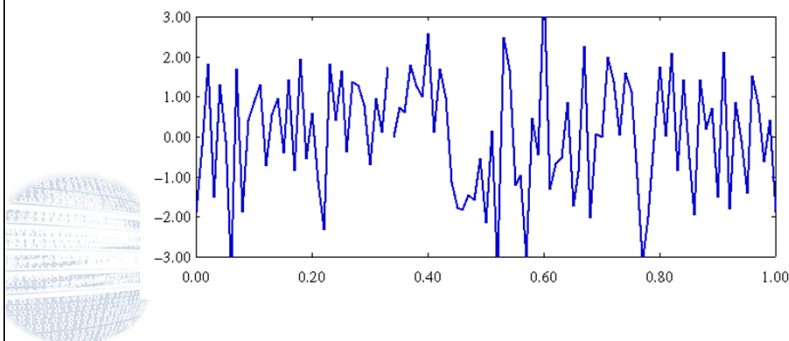


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Noise in Time Domain

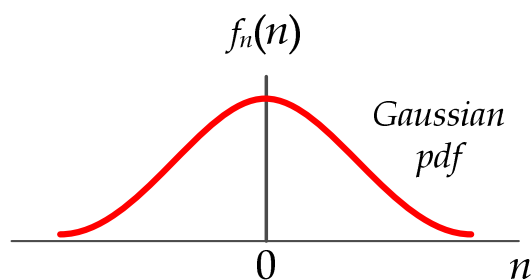
- Noise is a purely **random** signal.
- Cannot be written as a deterministic equation.



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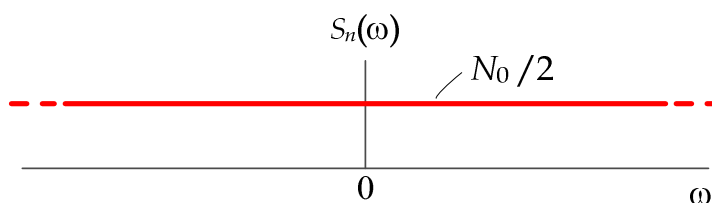
Noise in Time Domain



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Noise in Frequency Domain



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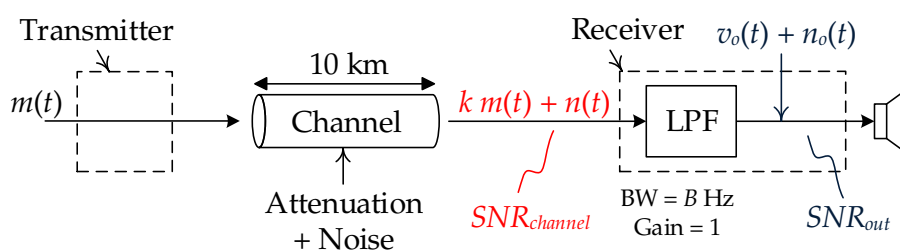
Example

- A voice signal $m(t)$ is transmitted without modulation through a 10 km long baseband channel with AWGN noise.
Assume:
 - Average power in $m(t)$ at the TX is 1 kW
 - Channel Attenuation = -5 dB/km
 - $S_n(\omega) = 2 \times 10^{-9}$ W/Hz = $N_0 / 2$
- Show the block diagram of the receiver.
- Determine $SNR_{channel}$ and SNR_{out} .

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Solution



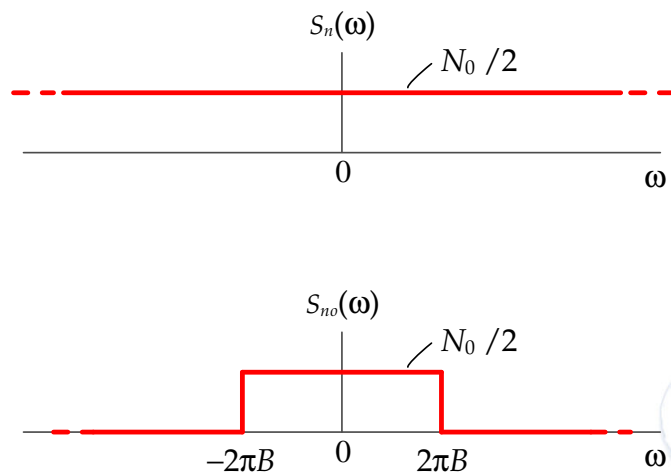
$$SNR_{channel} = \frac{k^2 \overline{m^2(t)}}{\overline{n^2(t)}}$$

$$SNR_{out} = \frac{\overline{v_o^2(t)}}{\overline{n_o^2(t)}}$$

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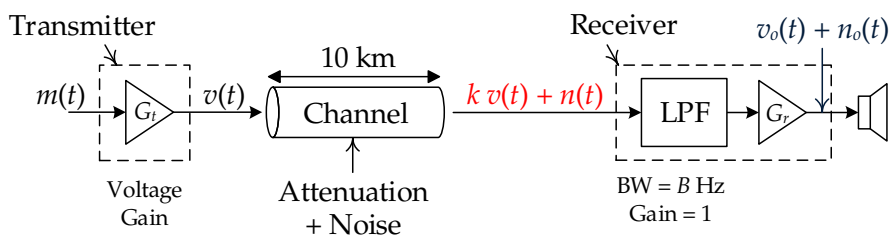
Band-Limited Noise



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Homework



- Is the gain G_r useful for improving quality?
- Is the gain G_r useful for anything else?
- Determine the gain G_t at the transmitter to get good quality voice at the receiver output.
- Or determine the necessary cooling (temperature) to get good quality voice.
- Or determine the channel attenuation to get good quality voice.

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Example 2

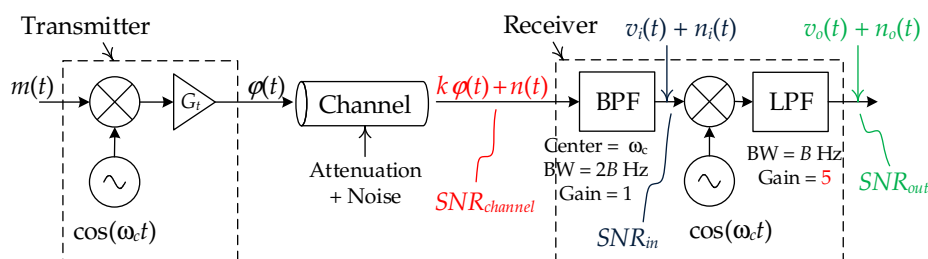
- A DSB-SC signal is sent through a channel with attenuation. The channel is affected by AWGN noise. At the receiver side:
- Show the block diagram of the receiver.
- Determine $SNR_{channel}$.
- Determine SNR_{in} .
- Determine SNR_{out} .
- Determine NF for the demodulator.



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Solution

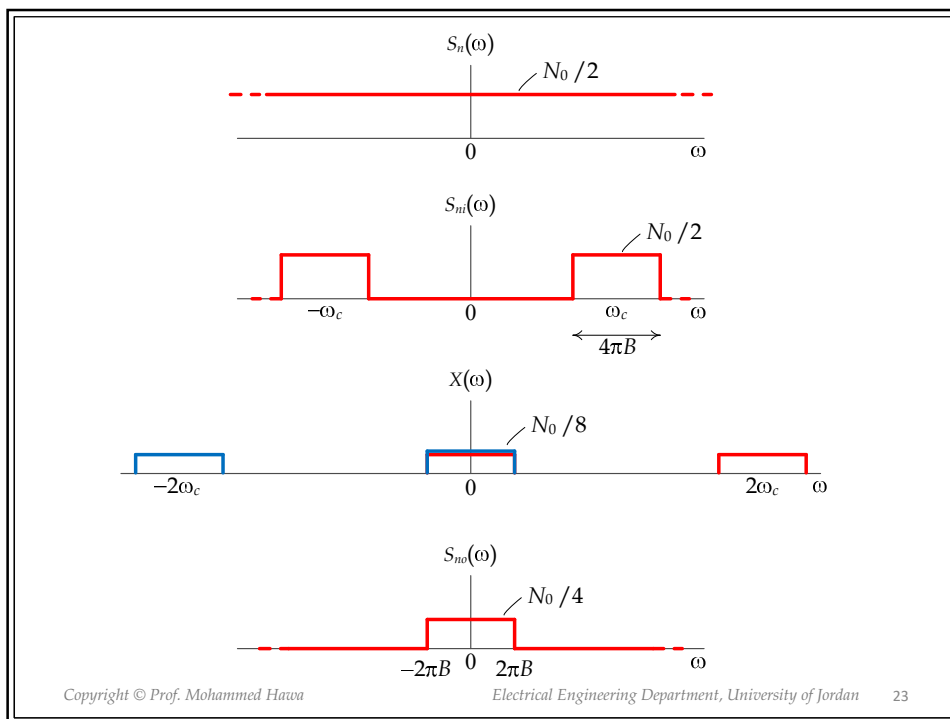


$$SNR_{channel} = 0, \quad SNR_{in} = \frac{k^2 \overline{m^2(t)}}{4N_0B}, \quad SNR_{out} = \frac{k^2 \overline{m^2(t)}}{2N_0B}$$

$$SNR_{channel} = -\infty \text{ dB}, \quad NF = -3 \text{ dB}, \quad SNR_{out} = \frac{S_{in}}{N_0B}$$

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Noise Factor

- Measures *degradation* of the quality (SNR) caused by components in the device.
- Noise factor is the ratio of SNR at the input to SNR at the output.

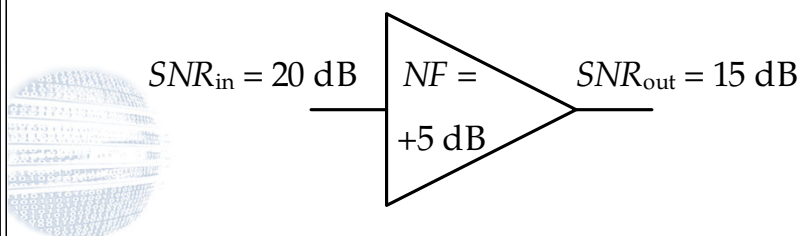
$$\text{Noise Factor} \triangleq \frac{SNR_{in} \text{ (unitless)}}{SNR_{out} \text{ (unitless)}}$$



Noise Figure, NF

- Noise Figure (NF) is Noise Factor but expressed in dB:

$$NF \triangleq SNR_{in}(dB) - SNR_{out}(dB)$$



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LNB Specifications



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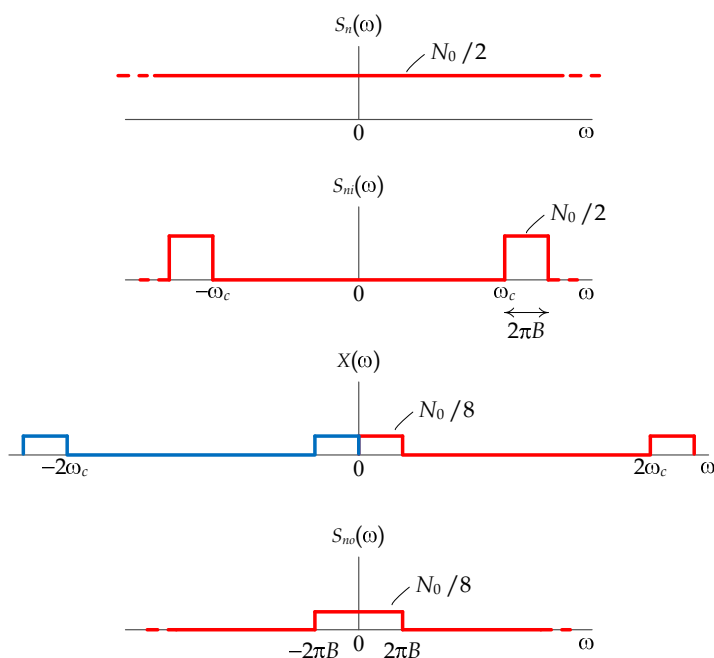
Homework 1

- A SSB-SC (USB) signal is sent through a channel with attenuation. The channel is affected by AWGN noise. At the receiver side:
- Show the block diagram of the receiver.
- Determine $\text{SNR}_{\text{channel}}$.
- Determine SNR_{in} .
- Determine SNR_{out} .
- Determine NF for the demodulator.



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Homework 2

- A QAM signal is sent through a channel with attenuation. The channel is affected by AWGN noise. Assume that $\overline{m_1^2} = \overline{m_2^2}$.
- Show the block diagram of the receiver.
- Determine SNR_{channel} .
- Determine SNR_{in} .
- Determine $SNR_{\text{out},1}$ and $SNR_{\text{out},2}$.
- Determine NF for the demodulator.



Modulation Technique	Modulated Signal Bandwidth	SNR_{out}	Noise Figure NF, dB	Typical Applications
DSB-SC	$2B$	$\frac{S_m}{N_0 B}$	-3	Analog instrumentation; multiplexing as part of FM stereo
SSB-SC	B	$\frac{S_m}{N_0 B}$	0	Point-to-point voice
VSB-SC	$B \sim 2B$	$\frac{S_m}{N_0 B}$	-3~0	Facsimile (Fax machines)
QAM	$2B$ for two signals	$\frac{S_{m, \text{effective}}}{N_0 B}$	0	Transmit color information in TV broadcasting; digital data
AM	$2B$	$\eta \frac{S_m}{N_0 B}$	$-10 \log(2\eta)$	Broadcast AM radio; point-to-point voice
SSB+C	B	$\eta \frac{S_m}{N_0 B}$	$-10 \log(\eta)$	Multiplexing in old telephony systems; point-to-point voice
VSB+C	$B \sim 2B$	$\eta \frac{S_m}{N_0 B}$	$-10 \log(2\eta) \sim -10 \log(\eta)$	Analog Television broadcasting
FM	$2\Delta f + 2B$	$\left(\frac{3\beta^2}{k_m^2}\right) \frac{S_m}{N_0 B}$	$10 \log\left(\frac{k_m^2}{6(\beta+1)\beta^2}\right)$	Broadcast FM radio; analog microwave links
PM	$2\Delta f + 2B$	$\left(\frac{(\Delta\theta)^2}{k_m^2}\right) \frac{S_m}{N_0 B}$	$10 \log\left(\frac{k_m^2 B}{2(\Delta\theta)^2(\Delta f + B)}\right)$	Telemetry; digital data